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APPLICATION OF THE REGULARIZATION METHOD TO DETERMINATION OF MULTILAYER STRATA PARAMETERS

M. Kh. Khairullin

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The problem of determining the collector properties of a multilayered petroleum stratum is among the class of inverse problems of underground hydromechanics; it is incorrectly formulated and nonlinear [1, 2]. Questions of the existence and uniqueness of the solution of this problem in the case of radial filtration in the presence of overflows through weakly permeable strata and infiltration were studied in [3]. The problem of determining the collector properties of a monostratum on the basis of the A. N. Tikhonov regularization method was considered in [4]. The present paper is its extension to the case of a multilayer stratum in the presence of overflows through weakly permeable connectors.

1. The majority of petroleum deposits has a laminar configuration due to features of the cumulative settling process. If the ratio of the permeability coefficients of two adjacent seams is less than 10^{-3} then the Myatiev–Girinskii scheme is applicable [1, 2]. We assume known the formulation of the direct problem in formulating the inverse problem. According to the Myatiev–Girinskii scheme the problem to determine the pressure fields $p_1 = p_1(x, y)$ and $p_2 = p_2(x, y)$ in a stratum with nonpermeable roof and floor, separated by a weakly permeable connector reduces under separate exploitation, to solving a system of partial differential equations in a multiconnected domain F with boundaries $\partial D = \Gamma + \sum_{k=1}^{m} \Gamma_k$ (Γ_k are circles of radius $r_s \cong 0.1$ m and centers at the points γ_k)

$$L_1p_1 + \omega(p_1 - p_2) = 0,\ L_1p_1 \equiv -\mathrm{div}(\sigma_1 \mathrm{grad}\, p_1),$$
$$L_2p_2 + \omega(p_2 - p_1) = 0,\ L_2p_2 \equiv -\mathrm{div}(\sigma_2 \mathrm{grad}\, p_2), \quad (1.1)$$

where σ_i, H_i ($i = 1, 2$) is the hydroconductivity coefficient and thickness of well-permeated seams, $\omega = \sigma_0/H_0^2$, σ_0, H_0 is the hydroconductivity coefficient and thickness of the weakly permeable connector, with the boundary conditions

$$\int_{\Gamma_l} \sigma_k \frac{\partial p_k}{\partial n} ds = q_{kl},\ \frac{\partial p_k}{\partial n}\Big|_{\Gamma_l} = 0,\ p_k|_\Gamma = 0,\ k = 1, 2,\ l = 1, 2, \ldots, m, \quad (1.2)$$

The second of the conditions (1.2) means that the pressure on the contour of each well is constant.

In operator form the boundary value problem (1.1) and (1.2) can be written in the form

$$Lp = 0,\ Mp = Q,\ Np = 0,\ p|_\Gamma = 0.$$

Here $p = (p_1, p_2)$; $L = \begin{pmatrix} L_1 + \omega E & -\omega E \\ -\omega E & L_2 + \omega E \end{pmatrix}$; $M = \{m_{kl}\}$, $N = \{n_{kl}\}$ are $2 \times m$ matrices with elements $m_{kl} = \int_{\Gamma_l} \sigma_k \frac{\partial}{\partial n} ds$, $n_{kl} = \frac{\partial}{\partial n}\Big|_{\Gamma_l}$ ($k = 1, 2$, $l = 1, 2, \ldots, m$); $Q = \{q_{kl}\}$ is the matrix of the debits.

The inverse problem is to find the quantities σ_0, σ_1, σ_2. Its initial data are the given debits $q_{k\ell}$, the values of the face pressure $\overset{*}{p}_{kl} = p_k|_{\Gamma_l}$ ($k = 1, 2$, $l = 1, 2, \ldots, m$, $m \geqslant 2$) and the pressure functions on the boundary of the filtration domain. This inverse problem generates a certain implicitly given nonlinear operator

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$$A\sigma = P^* \tag{1.3}$$

($\sigma = (\sigma_0, \sigma_1, \sigma_2)$, $P^* = \{p_{kl}\}$ is the matrix of the face pressures). The matrix P^* is ordinarily known inexactly: $||P^* - P_\delta^*|| \leqslant \delta$ ($||\cdot||$ is the norm in the Eulcidean space R^{2m}, and δ' is the error in measurement]. Solution of the operator equation (1.3) with the approximate right side is realized on the basis of minimizing the smoothing functional [5-7]

$$M^{\alpha}(\sigma) = ||A\sigma - P_\delta^*||^2 + \alpha\Omega(\sigma), \tag{1.4}$$

where $\Omega(\sigma) = \sum_{i=0}^{2} (\sigma_i - \sigma_i^0)^2$; $\alpha = \alpha(\delta)$ is the regularization parameter that agrees with the observation error.

Construction of an iteration process to minimize the smoothing functional (1.4) is performed by the scheme proposed in [6, p. 83]. Successive approximations σ^n are constructed in this manner: in the neighborhood of σ_n for a fixed value of the regularization parameter $\alpha = \alpha_n$ a nonlinear parameter $A\sigma$ is represented in the form

$$A\sigma = A\sigma^n + A_\sigma'(\sigma^n)(\sigma - \sigma^n) + O(||\sigma - \sigma^n||)$$

($A_\sigma'(\sigma^n)(\sigma - \sigma^n)$ is the Frechet differential], then the functional

$$M^{\alpha_n}(\sigma) = \| A\sigma^n + A_\sigma'(\sigma^n)(\sigma - \sigma^n) - P_\delta^* \|^2 + \alpha_n\Omega(\sigma)$$

becomes quadratic and its extremal is found from the Euler equation.

2. An explicit expression of the Frechet differential can be obtained by methods of perturbation theory [8]. Let $\sigma = \tilde{\sigma} + \delta\sigma$ ($\delta\sigma$ is a perturbation of the vector σ), $\tilde{p} = (\tilde{p}_1, p_2)$ is the solution of the boundary value problem

$$\tilde{L}\tilde{p} = 0,\ \tilde{M}\tilde{p} = Q,\ N\tilde{p} = 0,\ \tilde{p}|_\Gamma = 0$$

($\tilde{L}$ and $\tilde{M}$ are operators obtained from L and M by replacing σ by $\tilde{\sigma}$).

Let us consider the vector function $\tilde{p}_j^i = (\tilde{p}_{1j}^i, \tilde{p}_{2j}^i)$ $(i = 1, 2, j = 1, 2, \ldots, m)$, that are solutions of the boundary value problems

$$\tilde{L}\tilde{p}_j^i = 0; \tag{2.1}$$

$$\tilde{M}\tilde{p}_j^i = E_{ij},\ N\tilde{p}_j^i = 0,\ \tilde{p}_j^i|_\Gamma = 0, \tag{2.2}$$

where E_{ij} is a $2 \times m$ matrix for which the element at the intersection of the i-th row and the j-th column equals one while the remaining elements equal zero.

We call the boundary value problems (2.1) and (2.2) conjugate. It can be shown that

$$\tilde{p} = \sum_{k=1}^{m} \tilde{p}_k^1 q_{1k} + \sum_{k=1}^{m} \tilde{p}_k^2 q_{2k}. \tag{2.3}$$

Let us define the scalar product of the functions $a = a(x, y)$, $b = b(x, y)$ as

$$(a, b) = \int_D a(x, y)\, b(x, y)\, dx\, dy$$

and the vector functions $f = (f_1, f_2)$, $g = (g_1, g_2)$

$$\langle f, g \rangle = (f_1, g_1) + (f_2, g_2).$$

The following equalities hold

$$\langle Lp, \tilde{p}_j^i \rangle = \langle \delta Lp, \tilde{p}_j^i \rangle + \langle \tilde{L}p, \tilde{p}_j^i \rangle = 0,\ i = 1, 2 \tag{2.4}$$

($\delta L = L - \tilde{L}$). It is easy to see that

$$\langle \tilde{L}p, \tilde{p}_j^i \rangle = (\tilde{L}_1 p_1, \tilde{p}_{1j}^i) + \tilde{\omega}(p_1, \tilde{p}_{1j}^i) - \tilde{\omega}(p_2, \tilde{p}_{1j}^i) +$$
$$(\tilde{L}_2 p_2, \tilde{p}_{2j}^i) + \tilde{\omega}(p_2, \tilde{p}_{2j}^i) - \tilde{\omega}(p_1, \tilde{p}_{2j}^i),\ i = 1, 2 \quad (\tilde{\omega} = \tilde{\sigma}_0/H_0^2). \tag{2.5}$$

Applying the third Green's formula [9] to the component $(\tilde{L}_1 p_1, \tilde{p}_{1j}^1)$ and using (1.2), (2.2), and (2.3), we obtain

$$(\tilde{L}_1 p_1, \tilde{p}_{1j}^1) = (\tilde{L}_1 \tilde{p}_{1j}^1, p_1) - \sum_{k=1}^{m} \tilde{p}_{1jk}^1 \int_{\Gamma_k} \sigma_1 \frac{\partial p_1}{\partial n} ds + \sum_{k=1}^{m} p_{1k}^* \int_{\Gamma_k} \tilde{\sigma}_1 \frac{\partial \tilde{p}_{1j}^1}{\partial n} ds +$$

$$+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{1j}^1\delta\sigma_1\frac{\partial p_1}{\partial n}\,ds=(\tilde{L}_1\tilde{p}_{1j}^1,\,p_1)-\sum_{k=1}^{m}\tilde{p}_{1jk}^1q_{1k}+p_{1j}^*+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{1j}^1\delta\sigma_1\frac{\partial p_1}{\partial n}\,ds \tag{2.6}$$

$\left(p_{1j}^*=p_1|_{\Gamma_j},\ \tilde{p}_{1jk}^1=\tilde{p}_{1j}^1|_{\Gamma_k}\right)$. Analogously we find

$$(\tilde{L}_2p_2,\tilde{p}_{2j}^1)=(\tilde{L}_2\tilde{p}_{2j}^1,\,p_2)-\sum_{k=1}^{m}\tilde{p}_{2jk}^1q_{2k}+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{2j}^1\delta\sigma_2\frac{\partial p_2}{\partial n}\,ds \tag{2.7}$$

$\left(\tilde{p}_{2jk}^1=\tilde{p}_{2j}^1|_{\Gamma_k}\right)$. The quantities $\tilde{p}_{1jk}^1,\tilde{p}_{1jk}^2,\tilde{p}_{2jk}^1$ have the meaning of mutual influence coefficients. The following equalities are valid

$$\tilde{p}_{1jk}^1=\tilde{p}_{1kj}^1,\ \tilde{p}_{2jk}^1=\tilde{p}_{1kj}^2,\quad j,k=1,2,\ldots,m. \tag{2.8}$$

By using the boundary conditions (2.2) it is sufficient to represent the difference $\tilde{p}_{1jk}^1-\tilde{p}_{1kj}^1$ in the form

$$\tilde{p}_{1jk}^1-\tilde{p}_{1kj}^1=\sum_{l=1}^{m}\int_{\Gamma_l}\tilde{p}_{1j}^1\tilde{\sigma}_1\frac{\partial\tilde{p}_{1k}^1}{\partial n}\,ds-\sum_{l=1}^{m}\int_{\Gamma_l}\tilde{p}_{1k}^1\tilde{\sigma}_1\frac{\partial\tilde{p}_{1j}^1}{\partial n}\,ds+\sum_{l=1}^{m}\int_{\Gamma_l}\tilde{p}_{2j}^1\sigma_2\frac{\partial\tilde{p}_{2k}^1}{\partial n}\,ds-\sum_{l=1}^{m}\int_{\Gamma_l}\tilde{p}_{2k}^1\tilde{\sigma}_2\frac{\partial\tilde{p}_{2j}^1}{\partial n}\,ds,$$

and then to use the third Green's formula and (2.1) to prove (2.8). The second of equations (2.8) is proved analogously.

Substituting (2.6) and (2.7) into (2.5) and applying (2.3) and (2.8), we obtain

$$\langle\tilde{L}p,\tilde{p}_j^1\rangle=\langle\tilde{L}\tilde{p}_j^1,p\rangle+p_{1j}^*-\tilde{p}_{1j}^*+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{1j}^1\delta\sigma_1\frac{\partial p_1}{\partial n}\,ds+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{2j}^1\delta\sigma_2\frac{\partial p_2}{\partial n}\,ds. \tag{2.9}$$

Using (2.1) and (2.9), we find from (2.4)

$$\langle\delta Lp,\tilde{p}_j^i\rangle+p_{1j}^*-\tilde{p}_{1j}^*+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{1j}^1\delta\sigma_1\frac{\partial p_1}{\partial n}\,ds+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{2j}^1\delta\sigma_2\frac{\partial p_2}{\partial n}\,ds=0.$$

Now remarking that

$$\langle\delta Lp,\ \tilde{p}_j^i\rangle=(\delta L_1p_1,\tilde{p}_{1j}^i)+(\delta L_2p_2,\tilde{p}_{2j}^i)+\delta\omega\left[(p_1-p_2,\tilde{p}_{1j}^i)+(p_2-p_1,\tilde{p}_{2j}^i)\right]\qquad(\delta\omega=\delta\sigma_0/H_0^2,\ \delta L_i=L_i-\tilde{L}_i\qquad(i=1,2)),$$

we have

$$(\delta L_1p_1,\tilde{p}_{1j}^1)+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{1j}^1\delta\sigma_1\frac{\partial p_1}{\partial n}\,ds+(\delta L_2p_2,\tilde{p}_{2j}^1)+\sum_{k=1}^{m}\int_{\Gamma_k}\tilde{p}_{2j}^1\delta\sigma_2\frac{\partial p_2}{\partial n}\,ds+\delta\omega\left[(p_1-p_2,\tilde{p}_{1j}^1)+(p_2-p_1,\tilde{p}_{2j}^1)\right]=-(p_{1j}^*-\tilde{p}_{1j}^*).$$

Analogous relationships are also true for i = 2. There follows from these relationships and the first Green's formula

$$\langle\delta\sigma_1\ \mathrm{grad}\ p_1,\mathrm{grad}\,\tilde{p}_{1j}^i\rangle+\langle\delta\sigma_2\,\mathrm{grad}\ p_2,\mathrm{grad}\ \tilde{p}_{2j}^i\rangle+(\delta\sigma_0/H_0^2)\left[(p_1-p_2,\tilde{p}_{1j}^i)+(p_2-p_1,\tilde{p}_{2j}^i)\right]=-(p_{1j}^*-\tilde{p}_{1j}^*),\quad i=1,\ 2,\ j=1,2,\ldots,m. \tag{2.10}$$

The equalities (2.10) set for us a connection between the changes in the hydroconductivity coefficient and the changes of the face pressure. Replacing p_i by $\tilde{p}_i$ in (2.10), and assuming $A_{ij}=\langle\delta\sigma_1\,\mathrm{grad}\,\tilde{p}_1,\ \mathrm{grad}\,\tilde{p}_{1j}^i\rangle+\langle\delta\sigma_2\ \mathrm{grad}\,\tilde{p}_2,\ \mathrm{grad}\,\tilde{p}_{2j}^i\rangle+(\delta\sigma_0/H_0^2)[(\tilde{p}_1-\tilde{p}_2,\tilde{p}_{1j}^i)+(\tilde{p}_2-\tilde{p}_1,\tilde{p}_{2j}^i)]$, $i=$ 1, 2, j = 1, 2,...,m, we obtain a representation of the Frechet differential

$$A_\sigma'(\tilde{\sigma})(\sigma-\tilde{\sigma})=\begin{pmatrix}A_{11}&\ldots&A_{1m}\\A_{21}&\ldots&A_{2m}\end{pmatrix}.$$

3. Let us examine the results of a numerical experiment. A petroleum stratum is exploited by three walls: the permeability coefficients and thickness of the seams are: k_1 = 0.7 D, H_1 = 10 m, k_2 = 0.35 D, H_2 = 8 m, k_0 = 0.0001 D, H_0 = 1 m, the fluid viscosity is μ = 1 cP, $\sigma_i=k_iH_i/\mu$ $(i=0,1,2)$, $D=\{(x,y):0\leqslant x,y\leqslant 1000\text{ m}\}$. The well coordinates and their

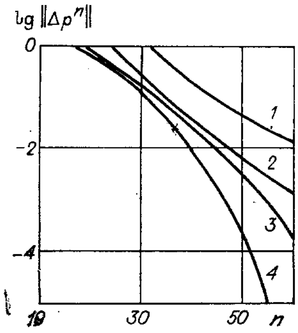

Fig. 1

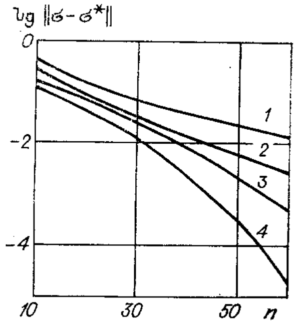

Fig. 2

debits are $\gamma_1 = (300,\ 300)$, $\gamma_2 = (500,\ 500)$, $\gamma_3 = (700,\ 700)$, $q_{11} = q_{21} = 160\ \mathrm{m^3/day}$, $q_{12} = q_{22} = 200\ \mathrm{m^3/day}$, $q_{13} = q_{23} = 240\ \mathrm{m^3/day}$. To obtain the numerical solution of the problem (1.1) and (1.2), finite differences are used, the well dimensions are ordinarily neglected by considering it a point source with power equal to the mass flow rate of a real well [10]. Computations were performed for an h = 100 m grid spacing in both variables. At each step of the iteration process, the residual in the face pressures $\|\Delta p^n\|^2 = \sum_{i=0}^{2}\sum_{j=1}^{3}(p_{ij}^{*n} - p_{ij}^{*})^2$, $n = 0, 1, \ldots$, is calculated, where p_{ij}^{*n} are values of the calculated face pressures, and $\|\sigma - \sigma^n\|^2 = \sum_{i=0}^{2}(\sigma_i - \sigma_i^n)^2$, $n = 0,\ 1, \ldots$. If $\|\Delta p^n\| \leqslant \delta$, then σ^n is an approximate solution of the problem (1.3). The regularization parameter is selected by means of the residual criterion.

Computation showed that the convergence of the iteration process depends on the selection of the initial approximation of the permeability coefficient of the connector and is independent of the initial approximation of the permeability coefficient of the quite permeable seams. In practice the selection of the initial approximation of the connector permeability coefficient is realized as follows. For different values of the connector permeability coefficient, 5-6 iterations are made, then that value is taken as the approximate value of the connector permeability coefficient for which the residual in the face pressures decreases most rapidly. Graphs of the functions $\log\|\Delta p^n\|^2$, $\log\|\sigma - \sigma^n\|$ as a function of the number of iterations n are presented in Figs. 1 and 2 for a different choice of the initial connector approximation, the curves 1-4 for $k_0^0 = 0.0005$, 0.00015, 0.00011, 0.0001 D, $k_1^0 = 0.5$ D, $k_2^0 = 0.5$ D. The permeability coefficients of the quite permeable seams are determined to 10^{-5} accuracy (curve 4) when the connector permeability coefficient is known, i.e., to the same accuracy as in [4]. If the connector permeability is known with error, then the accuracy of finding the permeability coefficients of the quite permeable seams depends on the accuracy of giving the connector permeability (curves 1-3).

The quantities q_{ij} and p_{ij}^* are measured with 1-3% accuracy in practice. Upon their insertion in the initial data of the problem the maximal error in determining the permeability coefficient in quite permeable seams is 4%, where the connector permeability in these computations was taken with a 10% error. Computations performed on model problems show that the proposed algorithm permits effective determination of the collector properties of petroleum strata.

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POSSIBILITIES FOR CONSTRUCTING A UNIFIED FAILURE THEORY

A. G. Ivanov

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The abundance of existing and newly developed materials and the various conditions for using them has led to creation of numerous, as a rule, semiempirical theories, criteria, concepts of failure, each of which holds for an experimentally studied range of change in parameters. These special theories together with previous experience of strength analysis have made it possible for a certain time to be limited to them. However, further development of technology in the direction of creating large unique objects intended for operating under conditions of intense dynamic loads, the impossibility in a number of cases of carrying out full-scale tests for these objects in order to explain their actual strength margins, and also continuing cases of unpredicted catastrophic failure for certain objects built in accordance with existing strength standards, require not so much development and creation of new failure criteria, as the requirement of finding a single physically substantiated approach to the problem as a whole, if only at the level of phenomenology without considering the fine details of failure phenomena and complicating circumstances. This theory with a capacity to some extent or other to combine special criteria (concepts) for failure should be built up taking account of the generally accepted fact, i.e., failure calculated for the whole in parts, is completion of work in proportion to the fracture surface. Therefore, work and energy specific for a unit of surface should act as criterial values. In fact, use of an energy approach with local consideration of conditions of a changeover of a crack to unsteady growth explains the vigorous development and success in understanding many details and features of brittle failure achieved by fracture mechanics (FM). Attempts to use FM for describing other forms of failure have been fruitful. However, it is not in a state of combining and describing all forms of it [1, 2].

Recently in works by the author with co-workers, and also by other domestic and overseas researchers a study has been carried out of failure for dynamically loaded shells. On the one hand these studies have made it possible to reveal a number of new effects not found in FM, and on the other hand, based on energy balance applied to the whole object in question or specified parts of it, to describe these phenomena and understand their physical nature. In future we call this the integral approach (IA) in contrast to the local approach used in FM. The integral approach makes it possible to look at the problem as a whole and to find a scheme for constructing an overall theory for failure. Previously such an attempt using the IA was made in [3]. Studies performed subsequently using the IA [3-14] provide a basis for its fruitfulness and necessity of developing it further.

We consider failure of a material cube with edge L stretched by forces σL^2 at two opposite faces. The rest of the faces are free. We also assume a piecewise linear rule for material deformation and it consists of an elastic region where

$$\sigma = \varepsilon E \tag{1}$$

up to $\sigma = \sigma_y$, where σ_y is yield stress (and elastic limit) for the material and region for plastic strain ($\sigma > \sigma_y$):

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